

# **Markscheme**

**May 2026**

**Mathematics: analysis and approaches**

**Higher level**

**Paper 2**

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## Instructions to Examiners

### Abbreviations

- M** Marks awarded for attempting to use a correct **Method**.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- R** Marks awarded for clear **Reasoning**.
- AG** Answer given in the question and so no marks are awarded.
- FT** Follow through. The practice of awarding marks, despite student errors in previous parts, for their correct methods/answers using incorrect results.

### Using the markscheme

#### 1 General

Award marks using the annotations as noted in the markscheme *eg M1, A2*.

#### 2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, *e.g. M1A1*, this usually means **M1** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **A3**, **M2** *etc.*, do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some students may be advantaged for that specific question item, it is likely that these students will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award **FT** marks as appropriate but do not award the final **A1** in the first part.

Examples:

|    | Correct answer seen | Further working seen                     | Any FT issues?                             | Action                                                                                      |
|----|---------------------|------------------------------------------|--------------------------------------------|---------------------------------------------------------------------------------------------|
| 1. | $8\sqrt{2}$         | 5.65685...<br>(incorrect decimal value)  | No.<br>Last part in question.              | Award <b>A1</b> for the final mark<br>(condone the incorrect further working)               |
| 2. | $\frac{35}{72}$     | 0.468111...<br>(incorrect decimal value) | Yes.<br>Value is used in subsequent parts. | Award <b>A0</b> for the final mark<br>(and full <b>FT</b> is available in subsequent parts) |

### 3 Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

### 4 Follow through marks (only applied after an error is made)

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then **FT** marks should be awarded for *their* correct answer, even when working is not present.

**For example:** following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen**. For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (e.g. probability greater than 1,  $\sin \theta = 1.5$ , non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that students may be using an incorrect value.
- If the student’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any **FT** marks in the subsequent parts. This includes when students fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these **FT** rules will be explicitly noted on the markscheme.
- If a student makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

## 5 Mis-read

If a student incorrectly copies values or information from the question, this is a mis-read (**MR**). A student should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread and do not award the first mark, even if this is an **M** mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (e.g. probability greater than 1,  $\sin \theta = 1.5$ , non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of students' own work does **not** constitute a misread, it is an error.
- If a student uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

## 6 Alternative methods

Students will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**.

## 7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, **M** marks and intermediate **A** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the student work is equivalent.

## 8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however students may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come “*from the use of 3 sf values*”.

**Simplification of final answers:** Students are advised to give final answers using good mathematical form. In general, for an **A** mark to be awarded, arithmetic should be completed, and any

values that lead to integers should be simplified; for example,  $\sqrt{\frac{25}{4}}$  should be written as  $\frac{5}{2}$ . An

exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example,  $\frac{10}{4}$  may be left in this form or written as  $\frac{5}{2}$ .

However,  $\frac{10}{5}$  should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g.  $4e^{2x} \times e^{3x}$  should be simplified to  $4e^{5x}$ , and  $4e^{2x} \times e^{3x} - e^{4x} \times e^x$  should be simplified to  $3e^{5x}$ . Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so  $x(x+1)$  and  $x^2 + x$  are both acceptable.

**Please note:** intermediate **A** marks do NOT need to be simplified.

## 9 Calculators

A GDC is required for this paper, but if you see work that suggests a student has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

## 10 Presentation of student work

**Crossed out work:** If a student has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the student indicates that they would like the work to be marked.

**More than one solution:** Where a student offers two or more different answers to the same question, an examiner should only mark the first response unless the student indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

## Section A

1. (a)  $a = -0.767$  ( $-0.767415\dots$ ),  $b = 130$  ( $130.345\dots$ ) **A1A1**  
**[2 marks]**
- (b)  $r = -0.996$  ( $-0.996248\dots$ ) **A1**  
**[1 mark]**
- (c) correct substitution of 79 into their regression equation **(M1)**  
 $69.7$  ( $69.7196\dots$ ) seconds **A1**  
**[2 marks]**
- (d)  $p = -1.29$  ( $-1.29331\dots$ ),  $q = 169$  ( $169.199\dots$ ) **A1**  
**[1 mark]**  
**Total [6 marks]**

2. (a) (i)  $(\tan 40^\circ =) \frac{h}{x}$  **A1**
- (ii)  $(\tan 15^\circ =) \frac{h}{70+x}$  **A1**  
**[2 marks]**
- (b) attempt to eliminate  $h$  or  $x$  from a system of 2 equations in 2 variables,  
 and form an equation in terms of one of these **(M1)**  
 $x \tan 40^\circ = (70+x) \tan 15^\circ$   
 $x = 32.8$  ( $32.8397\dots$ ) m **A1**  
**[2 marks]**
- (c)  $h = 32.839\dots \times \tan 40^\circ$  **(M1)**  
 $h = 27.6$  ( $27.5558\dots$ ) m **A1**  
**[2 marks]**  
**Total [6 marks]**

3. (a) 25 cubic meters A1  
[1 mark]

(b) substituting half their value of initial volume and  $t = 4$  into the equation (M1)

$$12.5 = 5 + 20e^{-4k}$$

$$k = 0.245 \text{ (0.245207...)} \text{ (accept } -\frac{1}{4} \ln \frac{15}{40} \text{ if solved using logs)} \quad \text{A1}$$

[2 marks]

(c) attempt to substitute their value of  $k$  into  $V$  and equate to 8 (M1)

$$8 = 5 + 20e^{-0.2452t}$$

$$t = 7.74 \text{ (7.73680...)} \quad \text{A1}$$

[2 marks]

Total [5 marks]

4. (a)  $\frac{1}{2}R^2\theta = 50$  OR  $100 = R^2\theta$  (A1)

$$2R + R\theta = 40 \text{ OR } R(2 + \theta) = 40 \text{ OR } \theta = \frac{40}{R} - 2 \quad \text{(A1)}$$

attempt to eliminate  $R$  or  $\theta$  to obtain an equation in a single variable (M1)

$$\frac{1}{2} \left( \frac{40}{\theta + 2} \right)^2 \theta = 50 \text{ OR } R^2 \left( \frac{40}{R} - 2 \right) = 100 \quad \text{(A1)}$$

$$\theta = 0.343145..., 11.6568...$$

$$\theta = 0.343 \text{ radians} \quad \text{A1}$$

[5 marks]

(b) substituting their  $\theta$  into  $\frac{1}{2}r^2\theta = 18$  (M1)

$$r = 10.2 \text{ (10.2426...)} \text{ cm} \quad \text{A1}$$

[2 marks]

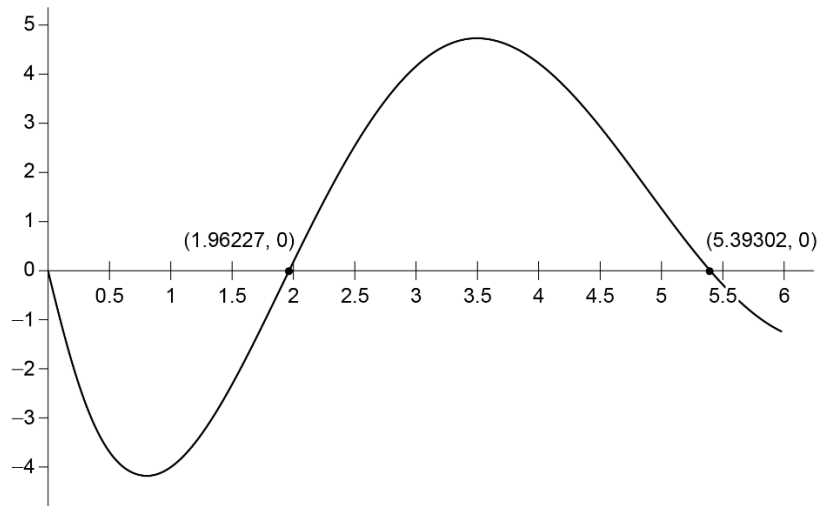
Total [7 marks]

5. (a) **METHOD 1**

recognising that the maximum displacement occurs where  $v(t) = 0$

**(M1)**

$t = 1.96227\dots$  OR  $t = 5.39302\dots$  OR one zero indicated on a sketch



finding a correct value of  $t$  where  $v(t) = 0$

**(A1)**

$t = 1.96227\dots$  OR  $t = 5.39302\dots$

attempt to find displacement between 0 and their  $t$ , which must be  $< 6$  seconds

**(M1)**

$$\int_0^{5.39302\dots} v(t) \, dt$$

$$= 4.81 \text{ (4.80592\dots) m}$$

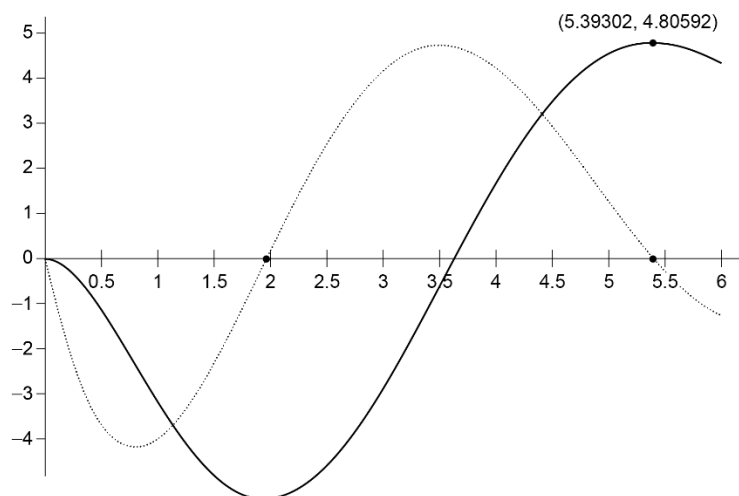
**A1**

**[4 marks]**

## METHOD 2

Sketch of graph of displacement

(M2)



attempt to find the maximum displacement (may be seen on sketch)

(M1)

$= 4.81 \text{ (} 4.80592 \dots \text{) m}$

A1

[4 marks]

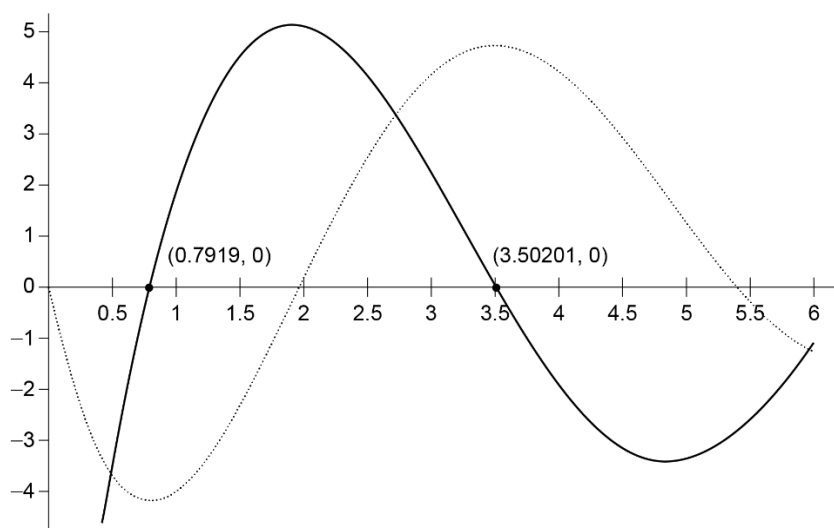
(b) attempt at a valid method for finding where acceleration is negative

(M1)

$v'(t) < 0$  OR  $0.791904 \dots$  OR  $3.50201 \dots$  OR

sketch of a graph of velocity indicating the maximum and minimum or stating that the gradient must be negative/downward sloping OR

sketch of a graph of acceleration, indicating zeros



$0.791904 \dots$  AND  $3.50201 \dots$

A1

$(0 \leq t < 0.792, 3.50 < t \leq 6)$

A1

[3 marks]

Total [7 marks]

6. (a)  $a = 3$

**A1**

**[1 mark]**

(b) attempt to express  $x^2$  (or  $x$ ) in terms of  $y$

**(M1)**

$$x^2 = 4 \left( 1 - \frac{y^2}{9} \right) \left( x = \sqrt{4 \left( 1 - \frac{y^2}{9} \right)} \right)$$

**(A1)**

forms a definite integral to form  $\pi \int_c^d x^2 dy$  with their expression for  $x^2$  in terms of  $y$

**(M1)**

$$V = \pi \int_0^3 4 \left( 1 - \frac{y^2}{9} \right) dy$$

**(A1)**

$$= 25.1 \text{ (25.1327..., } 8\pi)$$

**A1**

**[5 marks]**

**Total [6 marks]**

7. (a) recognition of the need to integrate  $f$

**(M1)**

$$\int_{0.5}^{1.5} f(x) dx$$

$$= 0.6875 \text{ OR } 0.688 \text{ OR } \left( \frac{11}{16} \right)$$

**A1**

**[2 marks]**

(b) (i)  $b = 0.6875 \text{ OR } 0.688 \text{ OR } \frac{11}{16}$

**A1**

$$(ii) \quad a = 0.0508 \left( 0.0507812..., \frac{13}{256} \right), c = 0.262 \left( 0.261718..., \frac{67}{256} \right)$$

**A1A1**

**[3 marks]**

(c) attempt to substitute their values into expected value formula

**(M1)**

$E(Y) = \text{their value of } b + 2 \times \text{their value of } c$

$$E(Y) = 1.21 \left( 1.21093..., \frac{155}{128} \right)$$

**A1**

**[2 marks]**

**Total [7 marks]**

8. (a) attempt to use Pythagoras theorem **M1**

$$15^2 = r^2 + (15 - h)^2$$

$225 = r^2 + h^2 - 30h + 225$  or an equivalent manipulation leading to the given answer **A1**

since  $r > 0$ ,  $r = \sqrt{30h - h^2}$  **AG**

**[2 marks]**

- (b) **METHOD 1**

recognize the use of chain rule (seen anywhere) **(M1)**

$$\frac{dV}{dt} = \frac{\pi}{3} \left( 90h \frac{dh}{dt} - 3h^2 \frac{dh}{dt} \right) \quad \text{A1}$$

$$\frac{dr}{dt} = \frac{1}{2} (30h - h^2)^{-\frac{1}{2}} \left( 30 \frac{dh}{dt} - 2h \frac{dh}{dt} \right) \quad \text{A1}$$

correct substitution of  $\frac{dV}{dt} = 20$  and  $h = 5$  to find  $\frac{dh}{dt}$  or  $\frac{dr}{dt}$  **(M1)**

$$\frac{dh}{dt} = 0.050929\dots, \quad \frac{dr}{dt} = 0.0455528\dots \text{ (exact values can be seen)}$$

$$\frac{dr}{dt} = 0.0456 \text{ cm s}^{-1} \left( \frac{8}{25\pi\sqrt{5}} \right) \quad \text{A1}$$

**METHOD 2**

recognize the use of chain rule (seen anywhere) **(M1)**

$$\frac{dr}{dt} = \frac{\frac{dV}{dt} \times \frac{dr}{dh}}{\frac{dV}{dh}} \text{ or equivalent}$$

attempt to find  $\frac{dV}{dh}$  or  $\frac{dr}{dh}$  at the point  $h = 5$  **(M1)**

$$\frac{dV}{dh} \Big|_{h=5} = 392.699\dots \quad \text{A1}$$

$$\frac{dr}{dh} \Big|_{h=5} = 0.894427\dots \quad \text{A1}$$

$$\frac{dr}{dt} = 0.0456 \text{ cm s}^{-1} \left( 0.0455528\dots, \frac{8}{25\pi\sqrt{5}} \right) \quad \text{A1}$$

**[5 marks]**

**Total [7 marks]**

9. (a) **METHOD 1**

number of ways of arranging boys between girls

$$= {}^8P_3 (=336) \quad \text{OR} \quad 3! \times {}^8C_3 (=6 \times 56) \quad (\text{A1})$$

arranging girls = 7!

$${}^8P_3 \times 7! (=336 \times 5040) \quad \text{OR} \quad 3! \times {}^8C_3 \times 7! (=6 \times 56 \times 5040) \quad (\text{A1})$$

$$= 1693440 \quad (\text{A1})$$

**METHOD 2**

number of ways of any two boys sitting next to each other  $6 \times 9!$  (A1)

this includes a duplicate of three boys sitting next to each other  $8! \times 3!$  (A1)

hence number of arrangements is  $10! - 6 \times 9! + 8! \times 3!$

$$= 1693440 \quad (\text{A1})$$

**METHOD 3**

number of ways of arranging 7 girls and a boy  $8!$  (A1)

number of ways of arranging remaining two boys  $7 \times 6$

hence number of arrangements is  $8! \times 7 \times 6 (=40320 \times 56)$  (A1)

$$= 1693440 \quad (\text{A1})$$

[3 marks]

(b) **METHOD 1**

number of ways of arranging boys at end, girls immediately next to them

$$= 3 \times 2 \times 7 \times 6 \quad (\text{A1})$$

arranging remaining people = 6!

$$3 \times 2 \times 7 \times 6 \times 6! \quad (\text{A1})$$

$$= 181440 \quad (\text{A1})$$

**METHOD 2**

number of ways of arranging boys, with two boys at the ends

$$= 3 \times 2 \times 6 \quad (\text{A1})$$

arranging girls = 7!

$$3 \times 2 \times 6 \times 7! \quad (\text{A1})$$

$$= 181440 \quad (\text{A1})$$

[3 marks]

Total [6 marks]

## Section B

10. (a) (i) use of inverse normal with 0.038 OR  $(1 - 0.073)$  OR 0.073 (M1)
- $z_1 = -1.77438...$  OR  $z_2 = 1.45380...$
- $\frac{20 - \mu}{\sigma} = -1.77438..., \frac{30 - \mu}{\sigma} = 1.45380...$  (or equivalent) (A1)(A1)
- attempt to solve their equations (must involve z-values, not probabilities) (M1)
- $\mu = 25.5$  (25.4965...) mins,  $\sigma = 3.10$  (3.09771...) mins A1
- (ii)  $P(15 < X < 22)$  (may be seen in shaded diagram) (M1)
- 0.129 (0.129152...) A1
- [7 marks]
- (b) (i) recognize the variable has a binomial distribution (M1)
- $Y \sim B(20, 0.826)$
- attempt to find  $P(Y \geq 13)$  (M1)
- 0.985 (0.985473...) A1
- (ii) recognize conditional probability in context (M1)
- $P(Y < 18 | Y \geq 13)$
- $= \frac{P(13 \leq Y \leq 17)}{P(Y \geq 13)}$  (A1)
- $= \frac{0.6872418...}{0.985473...}$  (A1)
- $= 0.697$  (0.697372...) A1
- [7 marks]
- continued...

Question 10 continued

- (c) recognize binomial distribution with unknown  $n$

**(M1)**

$$Y \sim B(n, 0.826)$$

| $n$ | $P(Y \leq 40)$ |
|-----|----------------|
| 41  | 0.999605...    |
| 42  | 0.996790...    |
| 43  | 0.986503...    |

**(A1)(A1)**

**Note:** Award the first **(A1)** for correct probability for  $n = 41$  or  $n = 42$  seen, then second **(A1)** if both  $n = 42$  and  $n = 43$  seen. Hence, award **(M1)(A1)A0A0** if only the correct probability for  $n = 41$  is seen.

$$n = 42$$

**A1**

**[4 marks]**

**Total [18 marks]**

11. (a)  $d = -3$

**A1**

**[1 mark]**

(b) attempt to expand their  $(x + d)(x + 7)$  or algebraic division

**OR** comparing coefficient in  $\frac{x^2 + bx + c}{x + d} = x + 7 + \frac{A}{x + d}$  where  $A \in \mathbb{R}$  **(M1)**

$b + 3 = 7$  **OR**  $x^2 + 4x - 21(+A = x^2 + bx + c)$  **A1**

$b = 4$  **AG**

**[2 marks]**

(c) **METHOD 1 finding  $f'(x)$  with product/quotient rule**

attempt to use quotient or product rule to find  $f'(x)$  **(M1)**

$\frac{(2x + 4)(x - 3) - (x^2 + 4x + c)}{(x - 3)^2}$  **OR**  $(2x + 4)(x - 3)^{-1} - (x^2 + 4x + c)(x - 3)^{-2}$  **(A1)**

attempt to solve their  $f'(-3) = 0$  **(M1)**

$c = 15$  **A1**

**METHOD 2 using oblique asymptote**

using  $f(x) = x + 7 + \frac{A}{x - 3}$ , where  $A \in \mathbb{R}$

$f'(x) = 1 - \frac{A}{(x - 3)^2}$  **(A1)(A1)**

**Note:** Award **A1** for each correct term. Award at most **A1A0** if additional terms are seen.

attempt to solve their  $f'(-3) = 0$  **(M1)**

$A = 36$

$c = -21 + A$

$c = 15$  **A1**

**[4 marks]**

continued...

Question 11 continued

- (d) two branches with approximately correct shape e.g. asymptotic

behaviour (for  $-30 \leq x \leq 30$ )

**A1**

asymptotes  $x = 3$  and  $y = x + 7$  drawn and labelled

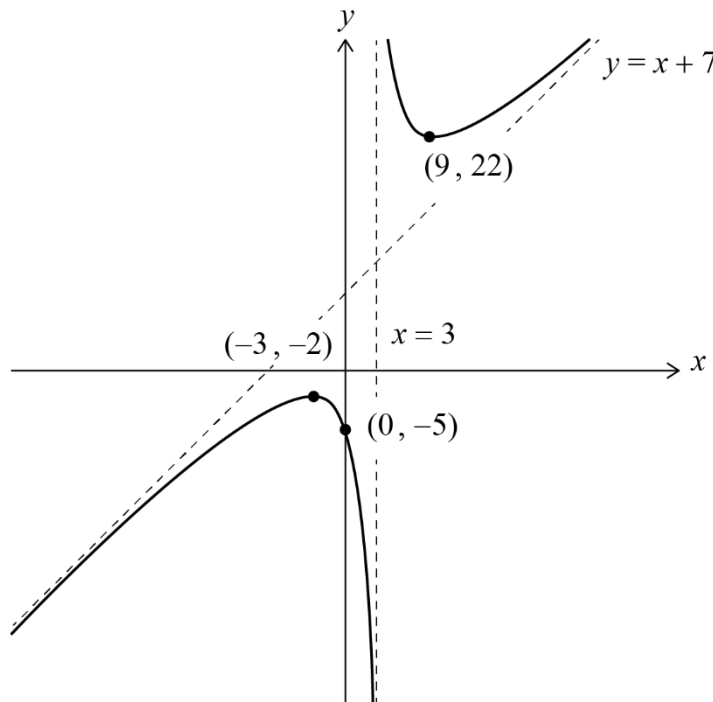
**A1**

correct coordinates for minimum,  $(9, 22)$ , labelled

**A1**

correct coordinates for maximum,  $(-3, -2)$  and y-intercept,  $(0, -5)$  or  $-5$ , labelled

**A1**



**Note:** The final 3 marks can be awarded independently, provided the approximate correct shape is seen for that branch e.g. it is possible to award **A0A0A0A1**.

**[4 marks]**

- (e)  $k > 22$

**A2**

**Note:** Award **A1** for 22 seen (e.g. with weak inequality, equality or nothing).

**[2 marks]**

continued...

Question 11 continued

(f) **METHOD 1**

$$g(x) = f(x-2) + 3 \quad \text{OR} \quad g(x) = \frac{(x-2)^2 + 4(x-2) + 15}{(x-2) - 3} + 3 \quad \text{OR}$$

$$g(x) = \frac{x^2 + 11}{x - 5} + 3 \quad \text{(A1)}$$

$$\text{attempt to solve } g(x) = 0 \quad \text{OR} \quad \frac{1}{g(x)} = \frac{x-5}{x^2 + 3x - 4} = \frac{x-5}{(x+4)(x-1)}$$

$$\text{OR sketch of } g(x) \quad \text{OR sketch of } \frac{1}{g(x)} \quad \text{(M1)}$$

$$x = -4 \text{ AND } x = 1 \quad \text{A1A1}$$

**METHOD 2**

$$\text{attempt to solve } f(x) = -3 \quad \text{OR sketch of } f(x) \text{ and } y = -3 \quad \text{(M1)}$$

$$-6 \text{ AND } -1 \quad \text{(A1)}$$

(horizontal translation of 2 units to the right)

$$x = -4 \text{ AND } x = 1 \quad \text{A1A1}$$

**[4 marks]**

**Total [17 marks]**

12. (a) attempt to use the recursive formula  $y_n = y_{n-1} + 0.01 \left( y_{n-1}^3 - \frac{y_{n-1}}{x_{n-1}} \right)$  (M1)

| $n$ | $x_n$ | $y_n$      |
|-----|-------|------------|
| 0   | 1     | 2          |
| 1   | 1.01  | 2.06       |
| 2   | 1.02  | 2.12702... |
| 3   | 1.03  | 2.20240... |
| 4   | 1.04  | 2.28784... |
| 5   | 1.05  | 2.38559... |

$y_1 = 2.06$  (A1)

$y_5 = 2.39$  A1

[3 marks]

- (b) attempt to use implicit differentiation with respect to  $x$  (M1)

$-\frac{2}{y^3} \frac{dy}{dx} = -2 \frac{dz}{dx}$  OR  $\frac{dy}{dx} = y^3 \frac{dz}{dx}$  (accept equivalent) A1

substitute  $\frac{dy}{dx}$  and  $y^2 = -\frac{1}{2z}$  into their derivative (M1)

$-\frac{2}{y^3} \left( y^3 - \frac{y}{x} \right) = -2 \frac{dz}{dx}$  OR  $y^3 - \frac{y}{x} = y^3 \frac{dz}{dx}$

$1 - \frac{1}{xy^2} = \frac{dz}{dx}$  A1

$\frac{dz}{dx} - \frac{2z}{x} = 1$  AG

[4 marks]

continued...

Question 12 continued

(c) **METHOD 1**

attempt to find integrating factor

**M1**

$$I(x) = e^{\int -\frac{2}{x} dx} = e^{-2\ln(x)}$$

$$I(x) = \frac{1}{x^2}$$

**A1**

$$\frac{1}{x^2} \frac{dz}{dx} - \frac{2z}{x^3} = \frac{1}{x^2}$$

$$\frac{z}{x^2} = \int \frac{1}{x^2} dx$$

**A1**

$$\frac{z}{x^2} = -\frac{1}{x} (+c)$$

**A1**

attempt to substitute  $x=1, z = -\frac{1}{2(2)^2}$  (or  $y=2$ ) into their integral to find  $c$

**(M1)**

$$c = \frac{7}{8}$$

substitute  $z = -\frac{1}{2y^2}$  (seen anywhere)

**(M1)**

$$-\frac{1}{2y^2x^2} = -\frac{1}{x} + \frac{7}{8}$$

$$\frac{1}{2y^2x^2} = \frac{8-7x}{8x}$$

$$y = \sqrt{\frac{4}{x(8-7x)}} \text{ or equivalent}$$

**A1**

continued...

Question 12 continued

**METHOD 2**

attempt to use the substitution  $z = vx$  and  $\frac{dz}{dx} = \frac{dy}{dx}x + v$  to reduce to a separable

equation

**M1**

$$\frac{dv}{dx}x + v - 2v = 1$$

$$\frac{dv}{dx}x = 1 + v$$

**A1**

$$\int \frac{1}{1+v} dv = \int \frac{1}{x} dx$$

**A1**

$$\ln|1+v| = \ln|x|(+c) \quad \text{OR} \quad \ln\left|1+\frac{z}{x}\right| = \ln|x|(+c)$$

**A1**

attempt to substitute  $x=1, z = -\frac{1}{2(2)^2}$  (or  $y=2$ ) into their integral to find  $c$

**(M1)**

$$c = \ln\left(\frac{7}{8}\right)$$

substitute  $z = -\frac{1}{2y^2}$  (seen anywhere)

**(M1)**

$$\ln\left|1 - \frac{1}{2y^2x}\right| = \ln\left(\frac{7}{8}x\right)$$

$$1 - \frac{1}{2y^2x} = \frac{7}{8}x$$

$$\frac{1}{y^2} = 2x\left(1 - \frac{7}{8}x\right)$$

$$y = \sqrt{\frac{4}{x(8-7x)}} \quad \text{or equivalent}$$

**A1**

**[7 marks]**

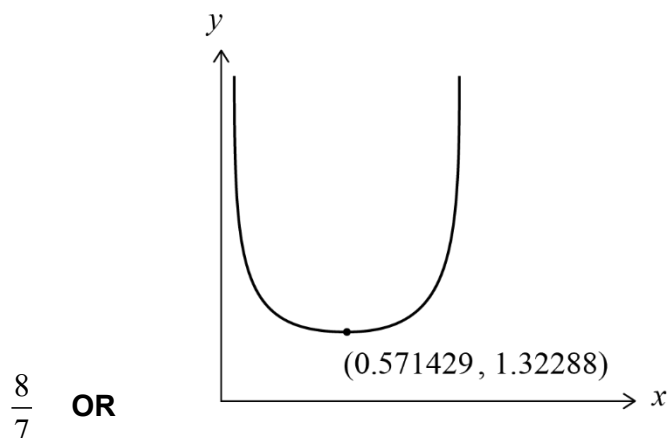
continued...

Question 12 continued

- (d) recognizing denominator is zero **OR** denominator cannot be zero

**OR** attempt to sketch the graph of  $y = \sqrt{\frac{4}{x(8-7x)}}$

**(M1)**



(domain)  $0 < x < \frac{8}{7}$  **OR**  $x \in \left]0, \frac{8}{7}\right[$

**A1**

1.32288...

(range)  $y \geq 1.32$  **OR**  $f(x) \geq 1.32$  **OR**  $f(x) \in [1.32, \infty[$

**A1**

allow exact value of  $\frac{\sqrt{7}}{2}$  in place of 1.32.

**Note:** The **A1** marks can be awarded independently.

**[3 marks]**

- (e) underestimates (when  $y = \sqrt{\frac{4}{x(8-7x)}}$ ) as it is concave-up

**R1**

**Note:** Award **R0** if candidate only states 'underestimates' without reference to concavity.

**[1 mark]**

**Total [18 marks]**